DETERMINATION OF AERODYNAMIC CHARACTERISTICS FOR PRESEPARATION DIFFUSERS BASED ON THE SOLUTION OF THE INVERSE BOUNDARY LAYER PROBLEM

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A finite-difference method is proposed for solving the inverse problem of the turbulent boundary layer of an incompressible fluid as applied to the calculation of geometric and aerodynamic characteristics for preseparation diffusers. The effect on these characteristics of the length of the transition area from a constant cross section channel to a preseparation diffuser is investigated. The reliability of the results of the calculation is confirmed by their satisfactory agreement with experimental data of various authors.

In spite of a great number of published experimental and theoretical investigations of diffuser channels, the problem of finding their optimal shapes remains urgent today. In 1930, L.G.Loitsyanskii first suggested that diffusers with the preseparation state of a boundary layer allow one to obtain the maximum pressure recovery for the given diffuser length. The separation criterion of the turbulent boundary layer and an expression for the computation of a flat preseparation diffuser contour were proposed in [1]. Later on Stratford theoretically [2] and experimentally [3] substantiated the possibility of producing a flow with zero surface friction along the full length. Hackenschmidt [4] on the basis of the Hagen criterion calculated the flat preseparation diffuser contour and experimentally verified its characteristics.

Systematic investigations of flat and axisymmetric preseparation diffusers based on the solution of the inverse boundary layer problem using the total momentum relation, the flow-rate equation, and the separation criterion in the form given by [1] were performed by A. S. Ginevskii and L. A. Bychkova [5]. The experimental verification of the results of these investigations is obtained in [6].

It must be noted that the application of one or another separation criterion for the design of preseparation diffusers inevitably leads to errors resulting from the approximate nature and insufficient generality of the separation criteria themselves. From this point of view, more universal is an approach based on the use of the inverse boundary layer problem, within whose framework the geometric and aerodynamic characteristics of the diffusers are determined directly from the coefficient of surface friction assigned along the contour (for preseparation diffusers its value is close to zero).

This approach is followed by A.S.Mazo [7]; however, the inverse problem for the boundary layer equations written for the entire flow region is solved by the author using an iterative solution of the direct problem with a refinement on each iteration of the sought value for the diffuser channel radius. A similar method is also employed in [8].

Besides, up to now, there have been practically no investigations of the problem of forming the transition area from a circular (flat) pipe contour to a preseparation diffuser contour. In performing the experimental investigations [6], a smoothing of the inlet section of the diffuser was made, and, in [7], a change of the surface friction coefficient from the inlet cross section (where $c_f \neq 0$) to a cross section starting from which $c_f = 0$ was set as a sine curve (a half-period).

One should also note the study [9] which optimized the shape of the diffuser, whose contour was determined by the specified surface friction coefficient in the form $\bar{c}_f = (1 - x)^N$. In the series of obtained diffusers, the condition $c_f = 0$ (the preseparation state of the boundary layer) was satisfied only at the inlet cross section.

In this paper we present the finite-difference method for the solution of the inverse boundary layer problem of an incompressible fluid with an assigned surface friction [10] as applied to flow calculations in preseparation diffusers. The geometric and aerodynamic characteristics of flat and axisymmetric preseparation diffusers are determined. The case of axisymmetric diffusers is used to investigate the influence of the length of the transition region from a circular pipe to a preseparation diffuser on its characteristics.

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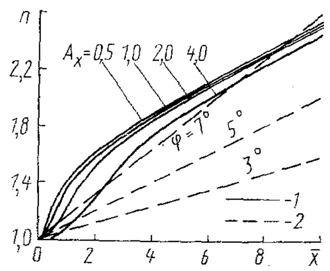


Fig. 1

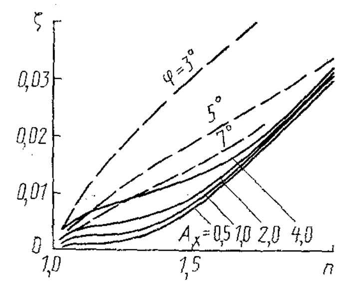


Fig. 2

Fig. 1. Expansion ratio of preseparation axisymmetric and conic diffusers ($Re = 2.42 \cdot 10^5$): (1) preseparation diffusers; (2) conic diffusers.

Fig. 2. Total-pressure losses in preseparation and conic diffusers (notation is the same as in Fig. 1).

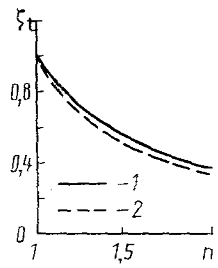


Fig. 3

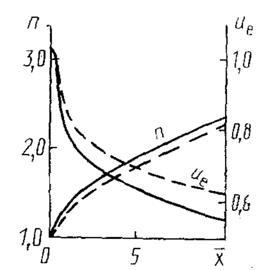


Fig. 4

Fig. 3. Total losses of preseparation diffusers at two values of the initial nonuniformity of the flow ($Re = 2.42 \cdot 10^5$): (1) $\Delta_0^* = 0.05$; (2) $\Delta_0^* = 0.03$.

Fig. 4. Geometric and aerodynamic characteristics of axisymmetric preseparation diffusers at two values of the initial nonuniformity of the flow (notation is the same as in Fig. 3).

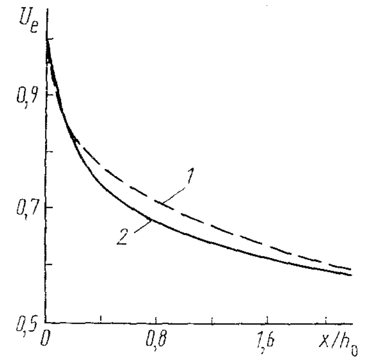
We consider the initial flow section within which no coupling of boundary layers occurs, i.e., a potential core exists. The change of the surface friction coefficient in the transition region is given by:

$$\bar{c}'_f = (1 + B\xi)(1 - \xi)^{B+d}, \tag{1}$$

where $\bar{c}_f = c_f/c_{f0}$, $\xi = x/A_x$, and B is an arbitrary constant (B = 3 was used in the calculation); also

$$d = -\frac{1}{c_{f_0}}\frac{dc_f}{d\bar{x}}\Big|_{\bar{x}=\bar{x}_0}. \tag{2}$$

It is obvious from Eq. (1) that, for $\xi = 0$, $c_f = c_{f0}$, where c_{f0} is the value of the surface friction coefficient at the inlet to the investigated transition region to the preseparation diffuser. Introducing the quantity d, which is determined by Eq. (2), into the exponent allows one to maintain a smooth joining of the function $\bar{c}_f(x)$ for the preinserted channel and the transition area.



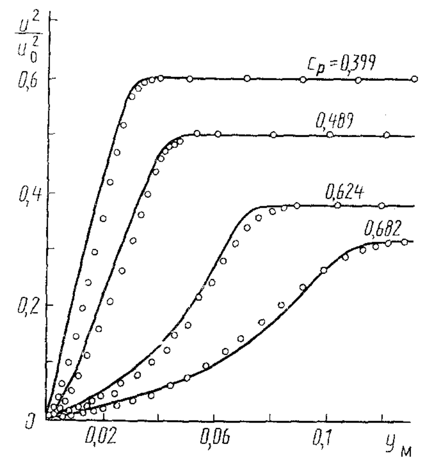


Fig. 5 Fig. 6

Fig. 5. Distribution of velocity in the core of the flow: (1) experiment of Stratford [3]; (2) calculation by the proposed method.

Fig. 6. Velocity head profiles; points indicate experimental data of Stratford [3]; the curves are for the values calculated by the proposed method.

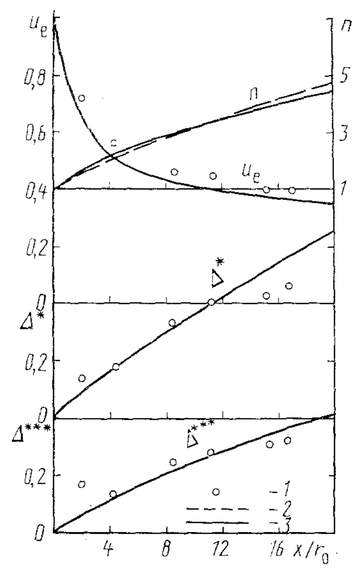


Fig. 7. Geometric and aerodynamic characteristics of an axisymmetric preseparation diffuser (Re = $1.4 \cdot 10^5$, $\Delta_0^* = 0.02$): (1) experiment of Bychkova [6]; (2) contour of the diffuser [6]; 3) calculation by the proposed method.

The value of the parameter A_x determining the transition region length is varied in the calculations.

The incompressible turbulent boundary layer equations are written in Crocco variables: $\bar{x} = x/L$ and $\bar{u} = u/U_e$ (here L is the characteristic linear dimension), by means of which the integration region becomes rectangular, and the density of the calculation grid is enhanced near the wall in changing to physical variables. Besides, Crocco variables, as we show below, prove to be convenient in formulating the inverse boundary layer problem with the assigned surface friction coefficient:

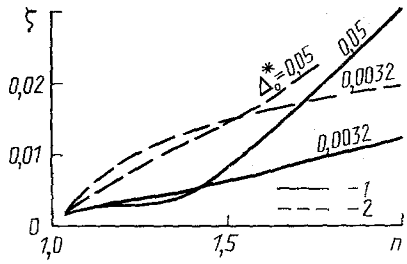


Fig. 8. Comparative efficiency of conic and axisymmetric preseparation diffusers with the calculated ($\Delta_0^* = 0.05$) and reduced ($\Delta_0^* = 0.0032$) degree of initial nonuniformity of the flow ($Re = 2.42 \cdot 10^5$): (1) preseparation diffuser; (2) conic diffuser ($\varphi = 7°$).

$$\frac{\partial \omega}{\partial \bar{x}} + A_1 \frac{\partial^2 (A_3 \omega)}{\partial \bar{u}^2} + A_2 \frac{\partial \omega}{\partial \bar{u}} + A_4 \omega = 0, \tag{3}$$

where

$$A_1 = -\frac{\omega^2}{\bar{u} r^j}; \quad A_2 = \alpha_e \frac{1 - \bar{u}^2}{\bar{u}};$$

$$A_3 = r^j \nu_\Sigma; \quad A_4 = \alpha_e - \left(\alpha_e \frac{1 - \bar{u}^2}{\bar{u}} \frac{1}{r} \frac{\partial r}{\partial \bar{u}} + \frac{1}{r} \frac{\partial r}{\partial \bar{x}}\right)^j;$$

$$\alpha_e = \frac{1}{U_e} \frac{dU_e}{d\bar{x}}; \quad \omega = \frac{\partial \bar{u}}{\partial \bar{y}}; \quad \bar{y} = \frac{y}{L}; \quad r = r_w - \bar{y} \cos\theta;$$

and $j = 0$ denotes the plane flow and $j = 1$ is for the axisymmetric one.

The boundary conditions are given by

$$\bar{u} = 1; \quad \omega = 0;$$

$$\bar{u} = 0: \quad \omega_w \frac{1}{r_w} \frac{\partial (r\bar{\tau})}{\partial \bar{u}} \bigg|_{\bar{u}=0} + \alpha_e = 0 \qquad (\bar{\tau} = \nu\omega). \tag{4}$$

The second condition of Eq. (4) can be replaced by the relation:

$$\bar{u} = 0: \; r_w \bar{\tau}_w = r\bar{\tau} + \alpha_e \bar{y}, \tag{5}$$

which is obtained by expanding the tangential stress in the vicinity of the wall in a Maclaurin series.

In Eq. (3) and in the boundary conditions (4) or (5), when solving the inverse boundary layer problem, besides the sought function $\omega(\bar{x}, \bar{y})$ the velocity at the outer edge of the boundary layer (in the core of the flow) U_e, entering in the complex α_e, is unknown. To calculate α_e, we employ an additional relation at the wall, which follows from the formulation of the inverse problem:

$$2\omega_w \nu = c_f(x) \tag{6}$$

and from one of the conditions (4) or (5).

Equations (3) are closed using the two-layer algebraic Van Driest-Clauser turbulent viscosity model:

$$\nu_T = \left\{0{,}4y \left[1 - \exp\left(-\frac{y}{26\nu} \sqrt{\frac{\tau}{\rho}}\right)\right]\right\}^2 \frac{\partial u}{\partial y}; \quad 0 \leqslant y \leqslant y_c;$$

$$\nu_T = 0{,}0168\delta^* U_e \left[1 + 5{,}5 \left(\frac{y}{\delta}\right)^6\right]^{-1}; \quad y_c < y \leqslant \delta, \tag{7}$$

where y_c is determined from the condition that the coefficient ν_T calculated by both formulas be smoothly joined.

The sought radius (height) of the diffuser cross section is found from a constant flow rate equation, which, for the initial section of the channel, has the form:

$$U_e = \frac{1}{n(1 - \Delta^*)}, \tag{8}$$

where $\Delta^* = 2\delta^*/r_w$.

The numerical solution of Eq. (3) is performed using a factorization method by an implicit difference scheme with the averaging of coefficients at the nodes of the calculation grid.

The internal losses in a diffuser are characterized by the total-pressure loss coefficient ζ, which is expressed in terms of parameters of the boundary layer at the end cross sections of the channel [5]:

$$\zeta = \frac{\Delta^{***}}{n^2(1-\Delta^*)^3} - \frac{\Delta_0^{***}}{(1-\Delta_0^*)^3}, \qquad (9)$$

here $\Delta^{***} = 2\delta^{***}/r_w$.

The total loss coefficient, including losses at the outlet from the channel, is defined as a sum of the coefficient ζ and a complementary loss coefficient $\Delta\zeta$:

$$\zeta_t = \zeta + \Delta\zeta, \qquad (10)$$
$$\Delta\zeta = \frac{1-\Delta^* - \Delta^{***}}{n^2(1-\Delta^*)^3}.$$

The calculations of the characteristics of preseparation axisymmetric diffusers were performed for the fixed value of $Re = 2.42\cdot 10^5$. An analysis of the results shows that, with a reduction in the transition segment length A_x, the expansion ratio n of the diffuser produced increases somewhat (Fig. 1), and the total pressure losses of the preseparation diffuser decrease at the fixed values of the expansion ratio (Fig. 2). The diffusers have a characteristic bell-shaped form at the initial section of flow. The curvature of generatrices decreases with increasing channel length, and near the cross section in which the linking of the boundary layers occurs, the diffuser shape is close to conic.

The total losses ζ_t for a given Re number depend solely on the expansion ratio of the diffuser and the initial nonuniformity of the flow Δ_0^* (Fig. 3). Decreasing Δ_0^*, which corresponds to decreasing the preinserted portion of a circular pipe, increases the efficiency of preseparation diffusers (Fig. 4).

To compare the parameters of the obtained preseparation diffusers with conic ones, the calculations were performed for the conic diffusers with the apex angles φ = 3,5, and 7°. The results of the calculations presented in Fig. 2 confirm the conclusion made in [5] that for similar values of the expansion ratio n and the initial nonuniformity Δ_0^* preseparation diffusers have smaller internal losses and substantially smaller axial dimensions. Preseparation diffusers permit attaining significant expansion ratios without a sharp increase in losses due to the separation of a flow, which takes place in conic diffusers with an increasing apex angle.

The reliability of the results of the calculation of preseparation diffusers by the proposed method is verified by comparing these results with experimental data. Figures 5 and 6 give this comparison with the results obtained by Stratford [3] both for the velocity in the core of a flow and for the velocity profile.

The dashed curve $n(\bar{x})$ in Fig. 7 corresponds to the shape of the diffuser which was used in the experiment of L.A.Bychkova [6], and the solid curve corresponds to the results of the calculation by the proposed method. Unfortunately, the data on the length of the transition region from a circular pipe to a preseparation diffuser and on the shape of this region are absent in [6]; however, such a comparison seems to be legitimate since the variation of the transition region length within the limits of A_x = 0.5-2.0, according to the above analysis, is accompanied by the variation of the values of n and U_e over a small range.

The basic drawback of preseparation diffusers is their single-mode behavior. Figure 8 compares the results of the calculation of the designed preseparation diffuser with those the conic one (φ = 7°) for the calculated and reduced degrees of nonuniformity of a flow at the inlet cross section. With decreasing Δ_0^*, the comparative efficiency of the preseparation diffuser, which is characterized by total-pressure losses, increases. Consequently, the design condition in determining the geometry of a preseparation diffuser is its operating condition for the highest value of Δ_0^*.

The results of the calculations show that the proposed method can be used in the design of preseparation diffusers whose application to various power plants will allow one not only to decrease their dimensions but also to substantially increase the efficiency of operation without additional expenditure of energy.

NOTATION

c_f, surface friction coefficient; r_w, radius of the cross section of an axisymmetric diffuser; h_w, channel height for a flat diffuser; $\bar{u} = u/U_e$, longitudinal velocity; U_e, velocity at the outer border of a boundary layer; $\nu_\Sigma = \nu + \nu_T$, effective viscosity; ν, coefficient of kinematic viscosity; ν_T, coefficient of turbulent viscosity; $\bar{x}$, $\bar{y}$ longitudinal and transverse coordinates; θ, angle between the diffuser axis and the tangent to the contour; δ, boundary layer thickness; δ^*, δ^{***}, displacement thickness and energy thickness of a boundary layer; n, expansion ratio of a diffuser: $n = (r_w/r_0)^2$ for axisymmetric diffusers, $n = h_w/h_0$ for flat diffusers; c_p, pressure coefficient; ρ, density; ζ, total-pressure loss coefficient; ζ_t, total loss coefficient; Re, Reynolds number; $\bar{\tau} = \tau/\rho U_e^2$ friction stress. Indices: 0, condition at the initial cross section; w, condition at the wall.

LITERATURE CITED

1. G. M. Bam-Zelikovich, Izv. Akad. Nauk SSSR, OTN, No. 12, 68-85 (1954).
2. B. S. Stratford, J. Fluid Mech., **5**, No. 1, 1-16 (1959).
3. B. S. Stratford, J. Fluid Mech., **5**, No. 1, 17-35 (1959).
4. M. Hakenschmidt, Maschinenbautechn., **13**, No. 10, 530-534 (1964).
5. A. S. Ginevskii and L. A. Bychkova, Heat and Mass Transfer [in Russian], Moscow (1968), Vol. 1, pp. 100-115.
6. L. A. Bychkova, Uch. Zap. TsAGI, **1**, No. 5, 89-93 (1970).
7. A. S. Mazo, Inzh.-Fiz. Zh., **44**, No. 5, 827-828 (1983).
8. A. S. Ginevskii, G. N. Emel'yanova, and A. V. Kolesnikov, Tr. TsIAM, No. 1093, 51-60 (1984).
9. G. Hokenson, J. Fluid Eng., **101**, 478-482 (1979).
10. V. M. Molochnikov, The Fifth All-Union Sci.-Pract. Conf. on Flight Safety, Extended Abstracts, p. 24, Leningrad (1988).